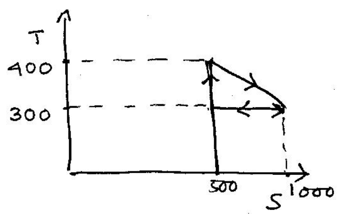
Physics II
ISI B.Math
Final Exam : November 15, 2019

Total Marks: 80
Time: 3 hours
Answer questions 1,2,3 and either 4 or 5



1. **(Marks: 8 + 12 = 20)**

a) Find the efficiency for a reversible engine operating around the cycle illustrated in the figure above. Entropy S is plotted along the x-axis in units J/K and absolute temperature T is plotted along the y-axis in degrees K.

b) A room air conditioner operates as a Carnot cycle refrigerator between an outside temperature T_h and a room at lower temperature T_i. The room gains heat from the outside at the rate $A(T_h - T_i)$; this heat is removed by the air conditioner. The power supplied to the cooling unit is P. Show that the steady state temperature of the room is

$$T_i = (T_h + P/2A) - [(T_h + P/2A)^2 - {T_h}^2]^{\frac{1}{2}}$$

2. **(Marks: 5 + 10 + 5 = 20)**

Consider a model system of N spins in equilibrium at temperature T. Each spin carries a magnetic moment that can assume the value $+m$ or $-m$. The energy of each spin in a magnetic field B is $-mB$ when the spin points along the magnetic field and mB when it points opposite to the magnetic field. Treat the spins as distinguishable.

(a) Find the partition function for the system

(b) Use the partition function to find an exact expression for the magnetization (the average magnetic moment per unit volume) M and the susceptibility $\chi = dM/dB$ as a function of the temperature, magnetic field and n, the particle density.
Show that the susceptibility is $\chi = \frac{nm^2}{T}$ in the limit $mB \ll T$.

(c) Find the Helmholz free energy and express the result as a function of only T and the parameter $x = \frac{M}{nm}$

3. **(Marks: 2 + 6 + 6 + 6 = 20)**

(a) State the fundamental postulate of equal a priori probabilities in statistical mechanics.

(b)The energy levels of a quantum mechanical oscillator with frequency ν is given by

$$\epsilon_n = (n + \frac{1}{2})h\nu$$

where $n = 0, 1, 2, \cdots$ and h is Planck's constant. When a system consisting of N independent oscillators has the total energy

$$E = \frac{1}{2}Nh\nu + Mh\nu$$

where M is an integer,

i) Find the number of states Ω_M corresponding to the energy E

ii)Find the entropy of the system as a function of the energy E in the large N approximation.

iii) Show that the energy E as a function of the temperature T of the system is given by

$$E = \frac{1}{2}Nh\nu \frac{\exp \frac{h\nu}{kT} + 1}{\exp \frac{h\nu}{kT} - 1}$$

You may use Stirling's approximation for large N : $\log N! \simeq N \log N - N$

4. **(Marks: 6 + 6 + 8 = 20)**

(a) In a Young's double slit experiment , one finds that by introducing a mica sheet of refractive index n to cover one of the slits, the central fringe now occupies the position previously occupied by the 10th bright fringe. If the source of light has wavelength λ , determine the thickness t of the mica sheet.

(b) The intensity of the light coming from one of the slits in a double slit arrangement is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.

(c) White light is incident normally on a thin film which has $n = 1.5$ and a thickness 5000 $\mathring{A}$. For what wavelengths in the visible spectrum (4000 - 7000 $\mathring{A}$) will the intensity of the reflected light be a maximum ?

5. **(Marks: 10 + 5 + 5 = 20)**

(a) Suppose a monochromatic and coherent source of light of wavelength λ passes through three parallel slits of equal width each separated by a distance d from its neighbour.The waves passing through each slit have the same amplitude and wavelength but have a constant phase difference $\phi = \frac{2\pi d \sin\theta}{\lambda}$ between waves from adjacent slits. If O is the position of the central slit,consider a point P on the screen such that OP makes an angle θ with the horizontal. Show that the intensity at P is given by

$$I = \frac{I_0}{9}\left[1 + 2\cos\phi\right]^2$$

Where I_0 is the maximum intensity associated with the principal maximum. Make a rough sketch of $\frac{I}{I_0}$ versus ϕ. Find the ratio of the intensities of the principal and secondary maxima.

(b) In a double slit arrangement where each slit is of width b and the separation between the slits is d. It is given that $d/b = 3$. Assuming that single slit diffraction effects cannot be neglected, find the orders of interference maxima that will be missing in the intensity pattern and explain why they will be missing .

(c)What is the state of polarization of a wave whose x and y components of the electric field are given by $E_x = E_0 \sin(\omega t + kz)$ and $E_y = E_0 \cos(\omega t + kz)$, where the symbols have their usual meanings?